# Applications of Inverse Methods in Room Acoustics

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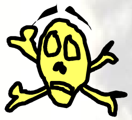
## Needing a new approach

**Questions emerging during the design:**

- **'What is the function of the room?'… OK**
- **'What are the acoustical objectives?'… OK**
- **'What are the architectural constraints?'… sad, but still right**



- **'What if we put a reflector here, an absorber there, a diffuser there, etc…?'**

**… instead:**

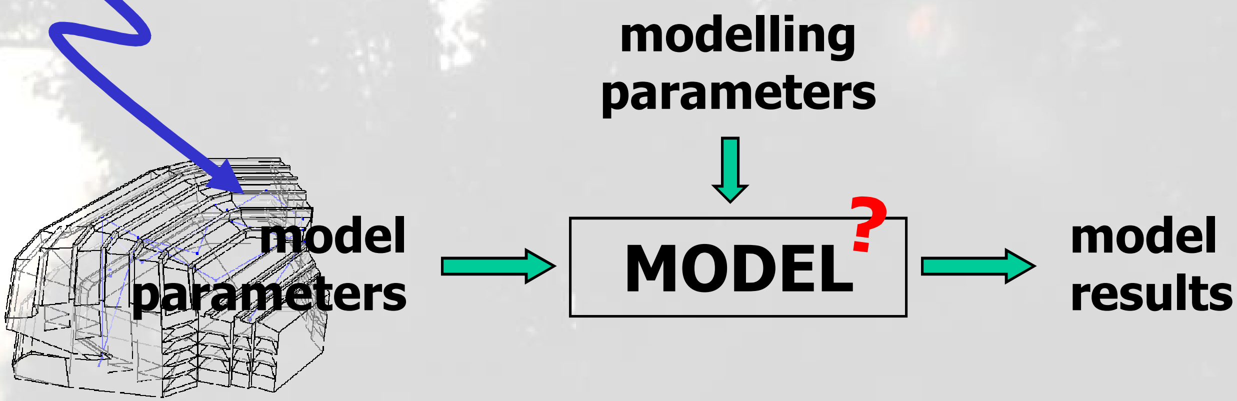
4. **'Is the acoustical objective feasible with the given conditions?'**
5. **'If not, what is causing the limitations?'**
6. **'If yes, which is the most/least critical area to treat/modify?'**

## Using room acoustical models

… in a trial-and-error approach requires:

- high computational power (or long deadlines),
- intuition and experience from the user,
- valid input…

## Some definitions…

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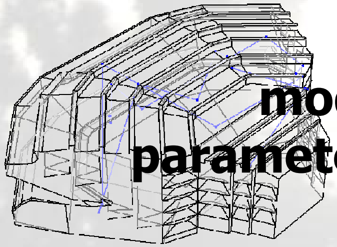
# What is the 'inverse' approach about?

modeling parameters



model parameters ⟷ ? **MODEL** ✓ ⟷ ? model results

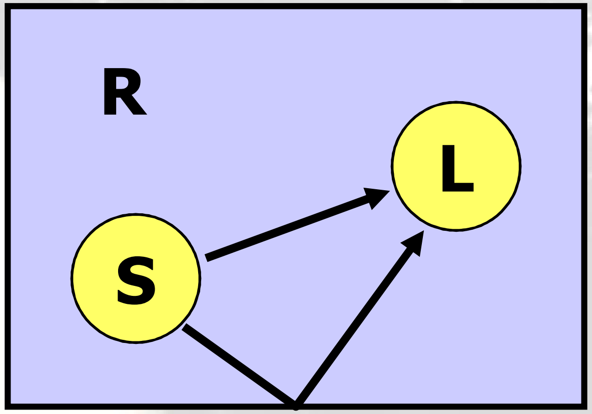
**Assumptions:**

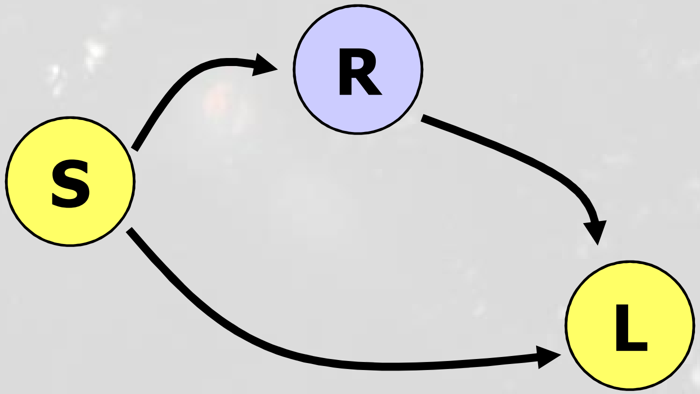
- the model is valid,
- there are analytical two-way connections.

## Theory and assumptions

**Let's assume the model is a valid energetic and geometric 'scan' of the possible energy transports between**

- **the source**
- **the room, and**
- **the listener.**



# Theory and assumptions (2)

## Pure specular energy transport equations:

$$
\begin{aligned}
(1-\alpha_1)^{m_{1,1}} \cdot (1-\alpha_2)^{m_{1,2}} \cdot \ldots \cdot (1-\alpha_N)^{m_{1,N}} &= e_1^* \\
(1-\alpha_1)^{m_{2,1}} \cdot (1-\alpha_2)^{m_{2,2}} \cdot \ldots \cdot (1-\alpha_N)^{m_{2,N}} &= e_2^* \\
&\vdots \\
(1-\alpha_1)^{m_{K,1}} \cdot (1-\alpha_2)^{m_{K,2}} \cdot \ldots \cdot (1-\alpha_N)^{m_{K,N}} &= e_K^*
\end{aligned}
$$

## Or…

$$
\begin{aligned}
m_{1,1}\log(1-\alpha_1) + m_{1,2}\log(1-\alpha_2) + \ldots + m_{1,N}\log(1-\alpha_N) &= \log e_1^* \\
m_{2,1}\log(1-\alpha_1) + m_{2,2}\log(1-\alpha_2) + \ldots + m_{2,N}\log(1-\alpha_N) &= \log e_2^* \\
&\vdots \\
m_{K,1}\log(1-\alpha_1) + m_{K,2}\log(1-\alpha_2) + \ldots + m_{K,N}\log(1-\alpha_N) &= \log e_K^*
\end{aligned}
\quad \Rightarrow \quad
\begin{aligned}
\mathbf{M}\log(1-\boldsymbol{\alpha}) &= \log \mathbf{e}^* \\
0 \le \boldsymbol{\alpha} &< 1 \\
0 \le \mathbf{e}^* &\le 1
\end{aligned}
$$

# Theory and assumptions (3)

**'Diffuse' and homogenous case: the Eyring formula**

$$m_i \cdot \log(1-\alpha) = \log(10^{-6}) \Rightarrow \frac{c \cdot T_{60}}{4 \cdot V/S} \log(1-\alpha) = -13.82$$

**Additional effects to take into account:**

- **air absorption (a),**
- **distance (d= $d_1$+$d_2$),**
- **listener visibility (V).**

**The reflectogram (detected energy paths):**

$$\log \mathbf{e} = \mathbf{V}\left[\log \mathbf{e}^* - \log \mathbf{a}(\mathbf{d}) - 2 \cdot \log(\mathbf{d}) - \log(4\pi)\right]$$

## Theory and assumptions (4)

**We have the equations to solve…**

**… but: K»N, the equation system is highly over-determined, optimization necessary.**

**Adding non-specular components (only SD combination):**

$$\begin{gathered}
\left[(1-\alpha_1)(1-\delta_1)\right]^{m_{1,1}} \cdot \left[(1-\alpha_2)(1-\delta_2)\right]^{m_{1,2}} \cdot \ldots \cdot \left[(1-\alpha_N)(1-\delta_N)\right]^{m_{1,N}} = e_1^* \\
\left[(1-\alpha_1)(1-\delta_1)\right]^{m_{2,1}} \cdot \left[(1-\alpha_2)(1-\delta_2)\right]^{m_{2,2}} \cdot \ldots \cdot \left[(1-\alpha_N)(1-\delta_N)\right]^{m_{2,N}} = e_2^* \\
\vdots \\
\left[(1-\alpha_1)(1-\delta_1)\right]^{m_{K,1}} \cdot \left[(1-\alpha_2)(1-\delta_2)\right]^{m_{K,2}} \cdot \ldots \cdot \left[(1-\alpha_N)(1-\delta_N)\right]^{m_{K,N}} = e_K^*
\end{gathered}$$

**Or…**

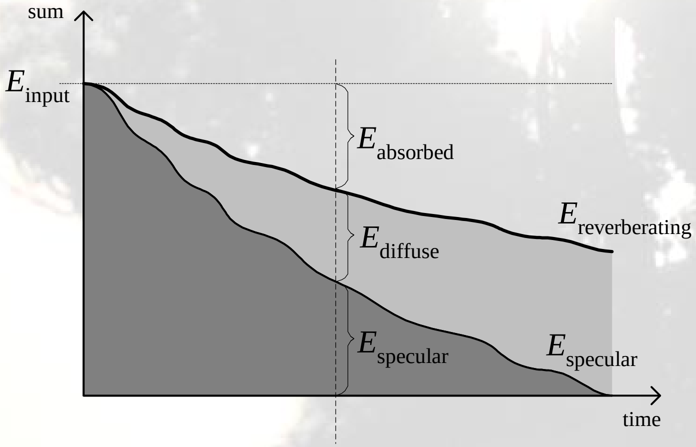
$$\mathbf{M}\log(1-\boldsymbol{\delta}) + \mathbf{M}\log(1-\boldsymbol{\alpha}) = \log \mathbf{e}^*$$

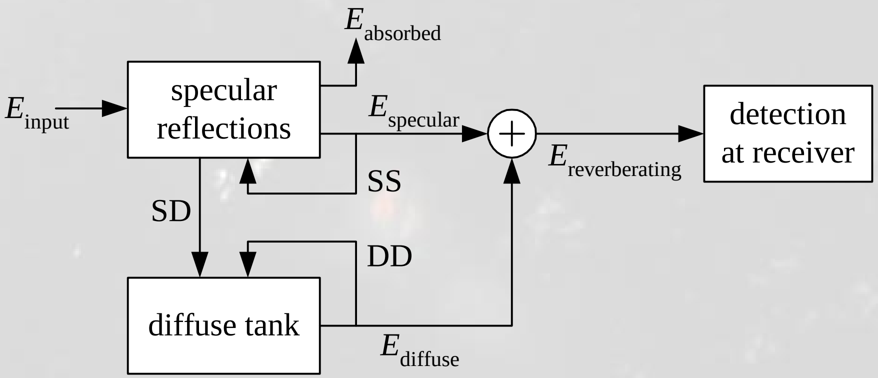
$$0 \le \boldsymbol{\alpha} < 1,\ 0 \le \boldsymbol{\delta} < 1,\ 0 \le \mathbf{e}^* \le 1$$

# Theory and assumptions (5)

**Only SS, SD combinations are handled, DS and DD are in a common 'diffuse tank'.**

**From the diffuse tank, diffuse energy is evenly distributed.**



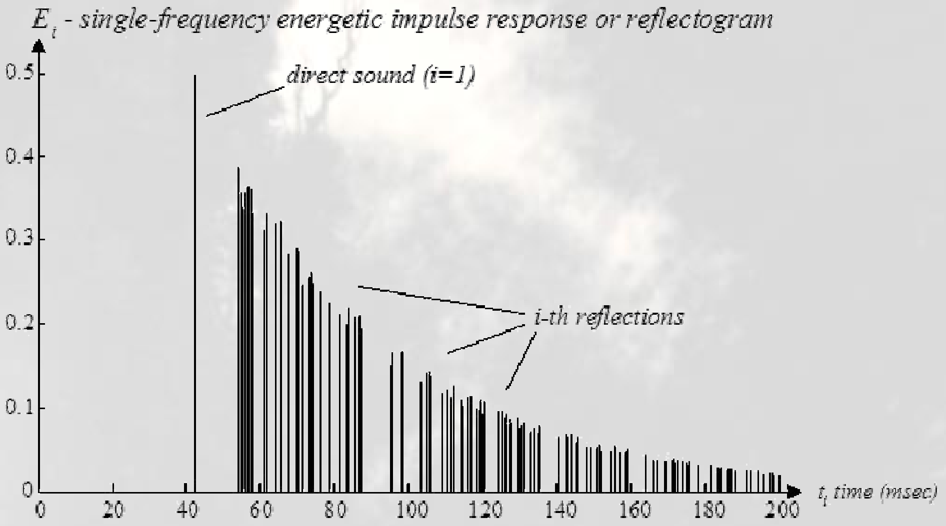

**Directional behavior of the source and the receiver can be also added.**

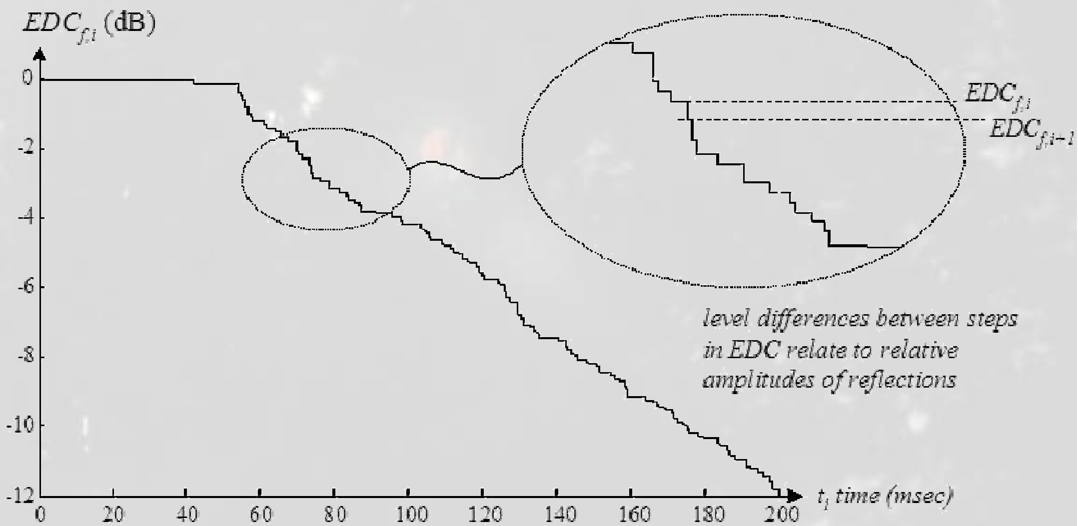
# Using the EDC

**The energy decay curve is:**

- **normalised in level**
- **directly related to room ac. parameters**

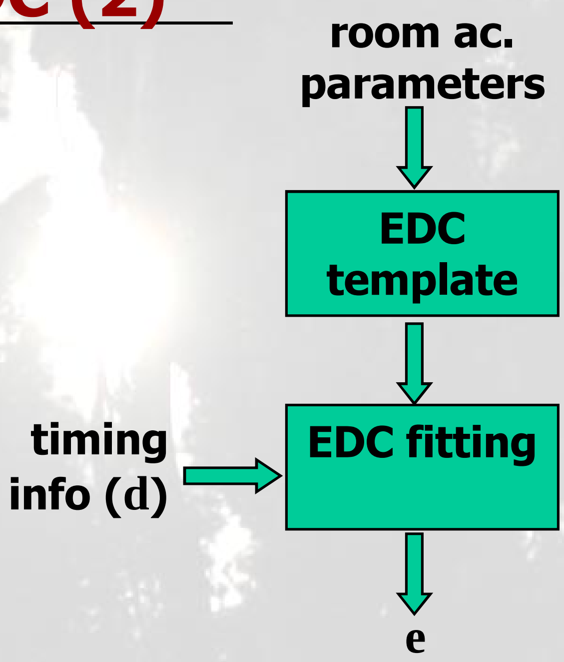
$$10^{EDC_{f,i}/10} - 10^{EDC_{f,i+1}/10} = \frac{e_i}{\sum_{j=1}^{\infty} e_j}$$



## Using the EDC (2)

**EDC fitting:**




$$10^{EDC_{f,i}/10} - 10^{EDC_{f,i+1}/10} = \frac{e_i}{\sum_{j=1}^{\infty} e_j}$$

**EDC template is feasible only if for each reflection:**

$$0 \leq \mathbf{e}^* \leq 1$$

**Otherwise one of the participating materials <u>cause the problem</u>.**

## Inverse method step-by-step

1. Create geometrical model with source-receiver positions
2. Run geometrical model (get M and V matrices and d vector)
3. Create (or measure) EDC template
4. Fit EDC with the known timings (use V matrix and d vector).
5. Get raw reflectogram (correct for directionality, air absorption, etc.)
6. Check if $0 \leq \mathbf{e}^{*} \leq 1$ and count for materials limiting feasibility.
7. If feasible, optimize to get absorption data.

## Inverse method applications

Feasibility: check whether a given EDC is feasible.

Analysis of possible EDCs: shows which materials limit feasibility.

Fitting to real measurements: measurement of model parameters.

## Experiences: benefits

Detailed and scaleable analysis without trial-and-error.

## Experiences: limitations

Convergence highly depends on real directionality and selected optimizations scheme.

Robust optimization available only for pure specular case.

## Current works

Optimization including diffuse part.

Optimizations for multiple source-receiver responses.

Inclusion of multi-channel parameters.

## Conclusions

Simple, flexible, effective model.

Investigations on the importance of the diffuse component.

Development of optimization algorithms (see current works).

The goal is still to gain a straightforward design methodology...

# Thank you for your attention!

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## Applications of Inverse Methods in Room Acoustics

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